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A voltage probe of the spin Hall effect

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Abstract
The spin Hall effect does not generally result in a transverse voltage. We predict that in systems with inhomogeneous electron density in the direction perpendicular to main current flow, the spin Hall effect is instead accompanied by a transverse voltage. We find that, unlike the ordinary Hall effect, this voltage is quadratic in the longitudinal electric field for a wide range of parameters accessible experimentally. We also predict spin accumulation in the bulk and sharp peaks of spin Hall induced charge accumulation near the edges. Our results can be readily tested experimentally, and would allow the electrical measurement of the spin Hall effect in non-magnetic systems and without injection of spin-polarized electrons.

(Some figures in this article are in colour only in the electronic version)

There is currently much interest in the spin Hall effect, which allows the polarization of electron spins without magnetic fields and/or magnetic materials [1–13, 15, 16]. In the spin Hall effect, electrically induced spin polarization accumulates near the edges of a channel and is zero in its central region. This effect is caused by deflection of carriers moving along an applied electric field by extrinsic [3] and/or intrinsic [5] mechanisms. In a non-magnetic homogeneous system, spin accumulation is *not* accompanied by a charge voltage because two spin Hall currents (due to spin-up and spin-down electrons) cancel each other [1]. The absence of transverse voltage leads to difficulties in probing the spin Hall effect: measuring a charge accumulation is much easier than measuring a spin accumulation. Recently, the spin Hall effect has been observed both optically [13–15] and electrically [16]. In the latter case, a charge accumulation has been created through injection of spin-polarized electrons into the sample [16].

In the present paper, we predict that in a system with an inhomogeneous electron density profile in the direction perpendicular to the direction of main current flow, the extrinsic spin Hall effect results in *both* spin and charge accumulations. The pattern of charge accumulation is determined by the interplay of two mechanisms. The first mechanism of charge accumulation is based on the dependence of spin-up and spin-down currents on local spin-up and spin-down densities. Spin currents, outgoing from regions with higher densities, are not fully compensated by incoming currents, therefore, a charge accumulation appears. This mechanism is primarily responsible for the non-zero Hall voltage. The second mechanism of charge accumulation is related to scattering of spin currents on sample boundaries which act like obstacles. Like in the case of Landauer resistivity dipoles [17], this scattering leads to formation of local charge accumulation, which is also expected in traditionally studied spin Hall systems. In addition, we show that in systems with inhomogeneous electron density the spin accumulation appears not only near the sample boundaries, but also in the bulk. Our proposal does not involve any use of magnetic materials and fields, therefore, the spin Hall effect can be measured electrically in completely non-magnetic systems and without injection of spin-polarized electrons.

To illustrate this effect, let us begin by considering a system having a step profile of electron density, as shown in figure 1. There are several possible ways to fabricate such a system including density depletion by an electrode, inhomogeneous doping [18], or variation of the sample height. What is important to us is that the perpendicular (in y direction) spin currents are different in the regions with different electron density. Then, if we consider currents passing through the boundary separating regions with different charge densities (n_1 and n_2), it is clear that the spin current from the region with higher electron density has a larger magnitude than the current in the reverse direction. The difference in currents implies charge transfer through the boundary and formation of a dipole layer.

Let us now provide a quantitative analysis of this effect. We employ a two-component drift-diffusion model [19, 20], and in order to find a self-consistent solution, we supplement

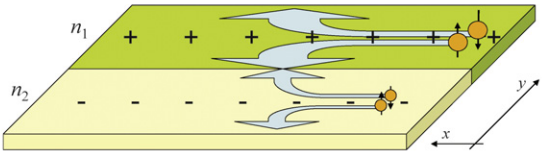

Figure 1. Spin Hall effect in a system with a step profile of the electron density in the y direction, $n_1 > n_2$. Spin currents through the boundary between n_1 and n_2 do not cancel each other, resulting in a transverse voltage.

the drift-diffusion equations with the Poisson equation. In our drift-diffusion calculation scheme, the inhomogeneous charge density profile $n(y)$ is found self-consistently for an assigned positive background density profile $N(y)$ (such as the one in figure 1), which, as discussed above, can be obtained in different ways. Assuming homogeneous charge and current densities in x direction and homogeneous x-component of the electric field in both x and y directions, we can write a set of equations including only y and t dependences:

$$e\frac{\partial n_{\uparrow(\downarrow)}}{\partial t} = \mathrm{div}\, j_{y,\uparrow(\downarrow)} + \frac{e}{2\tau_{\mathrm{sf}}}\left(n_{\downarrow(\uparrow)} - n_{\uparrow(\downarrow)}\right), \qquad (1)$$

$$j_{y,\uparrow(\downarrow)} = \sigma_{\uparrow(\downarrow)}E_y + eD\nabla n_{\uparrow(\downarrow)} \pm \gamma I_{x,\uparrow(\downarrow)}, \qquad (2)$$

and

$$\mathrm{div}\, E_y = \frac{e}{\varepsilon\varepsilon_0}\left(N(y) - n\right), \qquad (3)$$

where $-e$ is the electron charge, $n_{\uparrow(\downarrow)}$ is the density of spin-up (spin-down) electrons, $j_{y,\uparrow(\downarrow)}$ is the current density, τ_{sf} is the spin relaxation time, $\sigma_{\uparrow(\downarrow)} = en_{\uparrow(\downarrow)}\mu$ is the spin-up (spin-down) conductivity, μ is the mobility, D is the diffusion coefficient, ϵ is the permittivity of the bulk, and γ is the parameter describing deflection of spin-up (+) and spin-down (−) electrons. The current $I_{x,\uparrow(\downarrow)}$ in x-direction is coupled to the homogeneous electric field E_0 in the same direction as $I_{x,\uparrow(\downarrow)} = en_{\uparrow(\downarrow)}\mu E_0$. The last term in equation (2) is responsible for the spin Hall effect.

Equation (1) is the continuity relation that takes into account spin relaxation, equation (2) is the expression for the current in y direction which includes drift, diffusion and spin Hall effect components, and equation (3) is the Poisson equation. It is assumed that D, μ, τ_{sf} and γ are equal for spin-up and spin-down electrons[1]. In our model, as it follows from equation (2), the spin Hall correction to spin-up (spin-down) current (the last term in equation (2)) is simply proportional to the local spin-up (spin-down) density. All information about microscopic mechanisms for the spin Hall effect is therefore lumped in the parameter γ.

Combining equations (1) and (2) for different spin components we can get the following equations for electron density $n = n_\uparrow + n_\downarrow$ and spin density imbalance $P = n_\uparrow - n_\downarrow$:

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial y}\left[\mu n E_y + D\frac{\partial n}{\partial y} + \gamma P \mu E_0\right] \qquad (4)$$

[1] This is a good approximation for the range of parameters considered in this work.

and

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial y}\left[\mu P E_y + D\frac{\partial P}{\partial y} + \gamma n \mu E_0\right] - \frac{P}{\tau_{\mathrm{sf}}}. \qquad (5)$$

Analytical solution. Before solving equations (3)–(5) numerically, let us try to find analytical solutions in specific cases. This will help us in the discussion of the numerical results. An analytical steady-state solution of these equations can indeed be found for the case of exponential density profile in a system which is infinite in the y direction. Our numerical calculations indicate that in real systems, finite in y-direction, this analytical solution is realizable in the central part of the sample.

The structure of equations (3)–(5) allows us to select a solution in the form

$$n = N(y) = A\mathrm{e}^{\alpha y}, \qquad (6)$$

$$P = C\mathrm{e}^{\alpha y}, \qquad (7)$$

$$E_y = \mathrm{const}, \qquad (8)$$

where A, C and α are constants (A and α are assigned). This solution corresponds to constant spin polarization $p = P/n$. Substituting equations (6)–(8) into equations (4) and (5) (note that the Poisson equation (3) is automatically satisfied) we obtain

$$\mu E_y A + D\alpha A + \gamma\mu E_0 C = 0, \qquad (9)$$

$$\mu E_y \alpha C + D\alpha^2 C + \gamma\mu E_0 \alpha A - \frac{C}{\tau_{\mathrm{sf}}} = 0. \qquad (10)$$

From these equations, eliminating E_y, we find

$$C = \frac{-1 \pm \sqrt{1 + (2\tau_{\mathrm{sf}}\gamma\mu E_0\alpha)^2}}{2\tau_{\mathrm{sf}}\gamma\mu E_0\alpha}A. \qquad (11)$$

The physical solution corresponds to the + sign in equation (11). It can be easily verified that the solution given by equations (6)–(8), (11) corresponds to $j_y = 0$. Substituting equation (11) into equation (9) we finally get

$$E_y = -\frac{D}{\mu}\alpha - \frac{-1 + \sqrt{1 + (2\tau_{\mathrm{sf}}\gamma\mu E_0\alpha)^2}}{2\tau_{\mathrm{sf}}\mu\alpha}. \qquad (12)$$

The first term on the RHS of equation (12) is the built-in electric field countering the gradient of electron density. The second term on the RHS of equation (12) is the electric field needed to compensate the transverse current arising due to the spin Hall effect. If we now assume that the sample has a finite (but large) width L, then, E_y can be interpreted as due to charge accumulation near the edges, as in the ordinary Hall effect. The measurable transverse voltage is associated with the second term on the RHS of equation (12) and can be approximately written as

$$V_{\mathrm{H}} \simeq L\frac{-1 + \sqrt{1 + (2\tau_{\mathrm{sf}}\gamma\mu E_0\alpha)^2}}{2\tau_{\mathrm{sf}}\mu\alpha} \approx \begin{cases} L\tau_{\mathrm{sf}}\mu\alpha\gamma^2 E_0^2, & 2\tau_{\mathrm{sf}}\gamma\mu E_0\alpha \ll 1 \\ L\gamma E_0, & 2\tau_{\mathrm{sf}}\gamma\mu E_0\alpha \gg 1. \end{cases} \qquad (13)$$

From this equation we see that the transverse voltage is quadratic in E_0 for small values of the parameter $2\tau_{sf}\gamma\mu E_0\alpha$, and linear in E_0 for large values of this parameter. In fact, the quadratic dependence is quite unusual, since in the ordinary Hall effect the Hall voltage is linear in the longitudinal current. The reason for this unusual dependence can be understood as follows. The charge current in the y direction, determined by the difference of spin-up and spin-down currents, has a component (related to the last term in equation (2)) proportional to the spin density imbalance P times γE_0. At small values of $2\tau_{sf}\gamma\mu E_0\alpha$, the spin density imbalance is proportional to γE_0 itself. Therefore, the charge current and transverse voltage are quadratic in E_0. At large values of $2\tau_{sf}\gamma\mu E_0\alpha$, the spin density imbalance saturates and the current dependence on E_0 becomes linear. Another difference with respect to the ordinary Hall effect is that the polarity of the transverse voltage in the spin Hall effect is fixed by the geometry of the structure, and does not depend on the direction of the longitudinal current.

Let us now estimate the magnitude of $2\tau_{sf}\gamma\mu E_0\alpha$. Taking parameters related to experiments on GaAs ($\tau_{sf} = 10$ ns, $\gamma = 10^{-3}$ [6], $\mu = 8500$ cm^2 V^{-1} s^{-1}, $E_0 = 100$ V cm^{-1}, $\alpha = 2/L$, $L = 100$ μm), we find $2\tau_{sf}\gamma\mu E_0\alpha = 3.4 \times 10^{-3}$. Therefore, in experiments with GaAs, most likely, a quadratic voltage dependence on the longitudinal electric field can be observed.

Numerical solution. Equations (3)–(5) can be solved numerically for any reasonable form of $N(y)$. We choose for their simplicity (and possibility to be realized in practice) a step profile and an exponential profile. We solve these equations iteratively, starting with the electron density $n(y)$ close to $N(y)$ and $P(y)$ close to zero and recalculating $E_y(y)$ at each time step[2]. Once the steady-state solution is obtained, the transverse voltage as a function of E_0 is calculated as a change of the electrostatic potential across the sample.

Figure 2 shows distributions of the charge density and spin density imbalance in systems with a step (panel (a) of figure 2) and exponential (panel (b) of figure 2) background densities. The values of parameters used for these particular simulations were selected to be close to experimental conditions reported in [13]. However, we have tested the robustness of our predictions by solving equations (3)–(5) for different values of parameters, and found that the predicted transverse voltage should be measurable under a wide range of experimental parameters. Quite generally, the self-consistent charge density $n(y)$ is very close to the background density $N(y)$. Small deviations of $n(y)$ from $N(y)$ can be observed in regions with strong gradients of $N(y)$. In particular, we can notice that the step profile of electron density in figure 2(a) is smoothed out. Such a charge redistribution is related to the diffusion term in equation (4). The charge diffusion leads to the formation of a built-in electric field that equilibrates the charge diffusion.

We also find that the induced spin density imbalance P in systems with inhomogeneous electron densities shows some new features, in addition to the well-known spin accumulation near the edges. For instance, in figure 2(a), P has an additional peak around $y = 50$ μm. In figure 2(b), P is almost constant in the central region of the sample. In both cases, the physics of non-zero spin density imbalance is the same: the spin current incoming from the right is stronger than the spin current incoming from the left. We note that the integral spin density imbalance is always zero.

At $E_0 = 0$, the system is spin-unpolarized and there is no transverse voltage. When the longitudinal current is switched on, the electron charge redistributes, and the associated voltage appears. The change of electron density due to the spin Hall effect is presented in figure 3. The first interesting observation is that there is a strong charge accumulation near the edges followed by a charge depletion region. Another observation is that the total electron density in the left region of the samples ($y < 50$ μm) has increased and, correspondingly, the total electron density in the right region has decreased. This change of the electron distribution can be seen in figure 3. Therefore, the left part of the samples is charged negatively and the right part is charged positively, as schematically shown in figure 1.

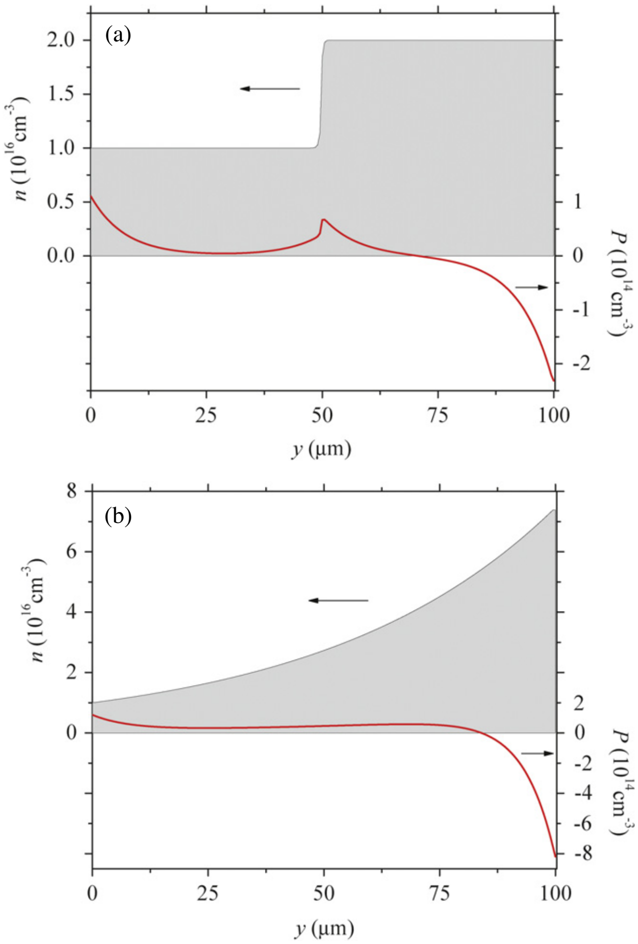

Figure 2. Distributions of the electron density $n(y)$ and spin density imbalance $P(y) = n_\uparrow - n_\downarrow$ for step (a) and exponential (b) background density profiles. The plots presented in the paper were obtained using the parameter values $\mu = 8500$ cm^2 V^{-1} s^{-1}, $D = 55$ cm^2 s^{-1}, $\varepsilon = 12.4$, $\tau_{sf} = 10$ ns, $\gamma = 10^{-3}$, $E_0 = 100$ V cm^{-1} and the background density profiles:
(a) $N = 10^{16}(1 + \theta(y - L/2))$ cm^{-3} and
(b) $N = 10^{16}\exp(2y/L)$ cm^{-3}, where $\theta(\cdot)$ is the step function, and $L = 100$ μm is the sample width.

[2] We have employed the Scharfetter–Gummel discretization scheme [21] to solve both equations (4) and (5) numerically.

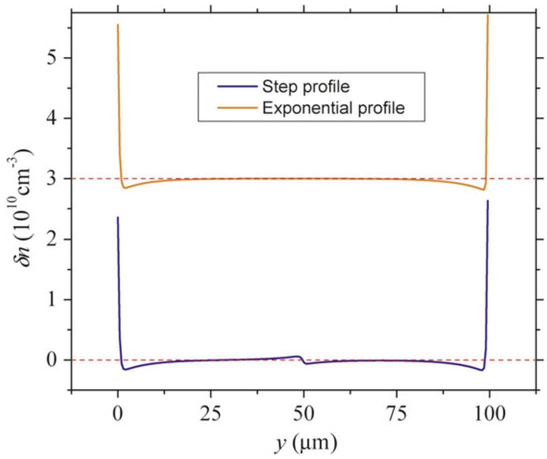

Figure 3. Variations in the transverse charge density induced by the longitudinal current. Here, $\delta n = n(E_0 = 100\ \mathrm{V\ cm^{-1}}) - n(E_0 = 0)$. The curve for the exponential profile has been shifted vertically by $3 \times 10^{10}\ \mathrm{cm^{-3}}$ for clarity. The dashed lines corresponding to $\delta n = 0$ are there to guide the eye.

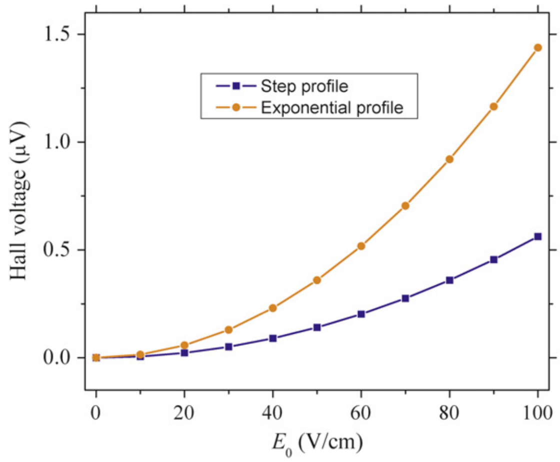

Figure 4. Transverse voltage as a function of the longitudinal electric field E_0.

The mechanism of formation of sharp peaks (of finite amplitude) of charge accumulation near the edges is similar to the mechanism of formation of Landauer resistivity dipoles [17]. From the point of view of spin currents, the sample edges act as obstacles which block the current flow, and lead to charge accumulation. The adjacent regions with the depleted electron density can be interpreted as screening clouds. We stress that this Landauer-type dipoles of charge accumulation are quite general for spin Hall systems, and should thus be present also in traditionally studied structures with a constant density profile.

We finally plot in figure 4 the change of the electrostatic potential across the sample as a function of longitudinal electric field. The voltage, for both density profiles, has a dependence on E_0 which is very close to the quadratic dependence we have predicted analytically in equation (13) for small values of $2\tau_{sf}\gamma\mu E_0\alpha$. The fact that this quadratic dependence appears also in the step profile, hints at a possible 'general' property of the transverse voltage in spin Hall systems with inhomogeneous densities. We emphasize that a transverse voltage should also appear in spin Hall systems with a homogeneous electron density, but inhomogeneous γ. This corresponds to the case in which the spin–orbit coupling is dependent on space [22, 23]. We also note that the ordinary potential scattering may contribute to the transverse voltage. However, the spin Hall contribution to the transverse voltage can be easily separated using its sensitivity to the in-plane magnetic field (via the magnetic field sensitivity of τ_{sf}).

In conclusion, we have shown that a transverse voltage would appear in spin Hall systems with inhomogeneous electron density in the direction perpendicular to main current flow. The striking result is that this voltage is generally quadratic in the longitudinal electric field, unlike the ordinary Hall voltage which is linear in the same field. These results can be easily verified experimentally, and would simplify tremendously the measurement of the spin Hall effect by allowing an electrical measurement of the latter in non-magnetic systems, and without injection of spin-polarized electrons.

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